

**TEACHING NOTE 09-02:  
UNDERSTANDING THE CASH FLOWS IN  
COLLATERALIZED DEBT OBLIGATIONS**

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The collateralized debt obligation or CDO is one of the more complex financial instruments in the markets today. There are numerous articles that describe the instrument, but reading a description and actually following where the money come from and where goes are two totally different things. In this note I will give a brief description of the instrument, but my focus is on a numerical example that will enable the reader to see the flow of the money. The example is simple and applies only to a specific and limited type of CDO, but once you see how the basic CDO works, it is not difficult to visualize the variations that exist in the market.

**What is a CDO?**

A collateralized debt obligation is a very general type of instrument that encompasses a broad family of instruments that are constructed through a process known as *securitization*. Securitization is a process of assembling a portfolio of financial instruments and selling claims on those instruments.<sup>1</sup> While that definition would fit a mutual fund and some other instruments, securitized instruments typically issue claims that have different characteristics. These differences arise from the fact that the returns from the underlying portfolio are altered before being passed through to the holders of the claims.<sup>2</sup> A mutual fund does not alter the returns of the underlying securities before passing them through to its shareholders. That is, mutual fund shareholders receive the returns on the stocks held in the fund. In addition, all mutual fund shareholders receive the same performance.

I believe the first type of securitized instrument was the mortgage-backed security. The simplest type of mortgage-backed security (MBS) is the mortgage pass-through, which does pass the payments right on through to the holders of the MBS.

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<sup>1</sup>The organizer of the securitized instrument typically forms a separate company for this specific purpose. Such companies formed for this purpose (and many others in business) are called *special purpose entities* (SPEs).

<sup>2</sup>At this point, you will probably start being confused, but be patient.

Thus, an MBS is not much different from a mutual fund. A variation of this instrument involves breaking the mortgage payments into their interest and principal components. These instruments are then called interest pass-throughs and principal pass-throughs. Thus, an investor in the MBS can buy only the interest or the principal payments from a portfolio of mortgages.<sup>3</sup>

The mortgages in the portfolio are often protected against credit risk, leaving essentially only two forms of risk: interest rate risk and prepayment risk.<sup>4</sup> Interest rate risk is the normal risk of changes in the value of a financial instrument as market interest rates go up and down and is common to every type of investment in a bond or fixed-income security. Prepayment risk is the risk of having the mortgage holders pay off their mortgages early. Prepayments are more common when interest rates have fallen because homeowners often refinance their mortgages with new mortgages at lower rates. When this happens, holders of mortgage-backed securities can suffer losses. For example, the holder of a principal pass-through has purchased a claim on the principal payments over the life of a series of mortgages. For simplicity, let us assume there is only one mortgage. If the borrower pays off the mortgage early, the stream of principal payments is terminated. Of course, the principal pass-through holder does receive the final principal payment but this occurs typically when interest rates have fallen. Thus, the money would have to be reinvested in a lower-rate environment. The holder of an interest pass-through loses by virtue of having the entire stream of interest payments terminated. Of course, this risk is built into the pricing of these instruments. It is when prepayments accelerate beyond what was expected that holders of mortgage-backed securities suffer the greatest losses.

The next stage in securitization came with the collateralized mortgage obligation or CMO. A collateralized instrument is typically structured to alter the stream of payments by selling claims that differ with respect to priority and risk.<sup>5</sup> These claims are broken up into what are called *tranches*. There are senior tranches, which are the lowest-risk claims, equity tranches, which are the highest-risk claims, and tranches that fall in

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<sup>3</sup>Mortgage-backed securities are usually either based on residential loans and are called Residential Mortgage-Backed Securities (RMBS) or commercial loans, which are called Commercial Mortgage-Backed Securities (CMBS).

<sup>4</sup>This protection is obtained by purchasing insurance policies against credit losses.

<sup>5</sup>This schedule of payments with their respective priorities is called the *waterfall*.

between with respect to risk. These are often called me mezzanine tranches. Within each of these three general categories there are tranches that differ with respect to priority of claims. Thus, an investor who buys a CMO typically buys one or more tranches. All of the tranches together make up the CMO.

After having much success with CMOs, financial institutions began putting together other types of debt instruments and hence, gave birth to the more general instrument, the CDO.<sup>6</sup> The concept of a CDO encompasses CBOs (collateralized bond obligations) and CLOs (collateralized loan obligations). In contrast to CMOs, where the greatest risk lies in the rate of prepayments, the greatest risk of a CDO is in the credit or default potential. CDOs are structured such that as defaults occur on the underlying loans, these losses are passed through to holders of the various tranches in a certain specified order. Equity holders bear losses first, while mezzanine holders bear losses second, and senior tranche holders bear losses last. As noted, each general category has sub-categories of tranches that bear losses in a scheduled order.

What a CDO does in essence is to take a portfolio of debt obligations and sell claims on them that differ with respect to the credit risk assumed by the holders of the claims. Consider a portfolio of loans held in a CDO and think of this portfolio of loans as having some average degree of credit risk. Holders of the senior tranches of the CDO will bear less risk than the average risk of the loans. Holders of the equity tranche will bear more risk than the average risk of the loans. This parceling of the risk enables investors who have different opinions about the risk of the loans to invest in the loans at a degree of risk of which they are comfortable.

### **A Numerical Example**

Consider a portfolio of three single-payment loans each with face value of \$1,000 and each paying off one period later.<sup>7</sup> The continuously compounded risk-free rate for the period is 4%. Call these loans A, B, and C. They have different degrees of credit risk

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<sup>6</sup>There is yet another family of related instruments, which is called asset-backed securities. This group generally includes structures that hold credit card or other types of receivables.

<sup>7</sup>The length of this period does not need to be specified for this example. We will start the example at the beginning of the period and end it at the end of the period. You can treat the period as one year. A single-payment loan means that an amount is borrowed today and the full amount plus interest is paid back later.

and default occurs, they have different recovery rates.<sup>8</sup> Let us assume that A has a recovery rate of 35%, B has a rate of 25%, and C has a rate of 15%. These numbers applied to the principal tell us how much would be paid if default occurs.

One of the most critical pieces of information in analyzing a credit-sensitive instrument is the probability of default. What complicates CDO analysis is that we must take into account the correlation of default. That is, if one instrument defaults, does it affect the likelihood of the other defaulting? We will assume that there are such relationships among these loans. The following information is given about the default correlations. Note that we do not specify the probability of B defaulting and the probability of C defaulting, though these can be obtained and we do so in the appendix.

<b>Event</b>	<b>Probability</b>
A defaults	5%
B defaults, given that A defaults	65%
B defaults, given that A does not default	7%
C defaults given that A defaults	55%
C defaults given that A does not default	4%

Notice that if A defaults, B is quite likely to default (65% chance), but if A does not default B is not likely to default (7% chance). Similar statements can be made about C.

Now let us set up a three-tranche CDO. The senior tranche has a loss limit of \$800, the mezzanine tranche has a loss limit of \$1,000, and the equity tranche has a loss limit of \$1,200. You can think of these amounts as the most that each tranche can lose. They are not the amounts invested, as we will determine shortly. We will now proceed to determine how much each tranche would cost to purchase.

In this example, there are eight possible outcomes as show below:

Outcome 1: Neither A, nor B, nor C default

Outcome 2: A defaults

Outcome 3: B defaults

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<sup>8</sup>A recovery rate is the percentage of the defaulted value that will be recovered if default occurs. Default does not automatically imply that the entire amount owed will not be paid.

Outcome 4: C defaults

Outcome 5: A and B default

Outcome 6: A and C default

Outcome 7: B and C default

Outcome 8: A, B, and C default

The table below shows these eight outcomes, the underlying payoffs, and the probabilities.

Outcome	Underlying Bond Payoffs				Probabilities of Outcomes			
	A	B	C	Total	A	B	C	Joint
1	\$1,000	\$1,000	\$1,000	\$3,000	95%	93%	96%	84.82%
2	\$350	\$1,000	\$1,000	\$2,350	5%	35%	45%	0.79%
3	\$1,000	\$250	\$1,000	\$2,250	95%	7%	96%	6.38%
4	\$1,000	\$1,000	\$150	\$2,150	95%	93%	4%	3.53%
5	\$350	\$250	\$1,000	\$1,600	5%	65%	45%	1.46%
6	\$350	\$1,000	\$150	\$1,500	5%	35%	55%	0.96%
7	\$1,000	\$250	\$150	\$1,400	95%	7%	4%	0.27%
8	\$350	\$250	\$150	\$750	5%	65%	55%	1.79%

Notice in each outcome for which A does not default, it pays \$1,000. A pays \$350 when it defaults, reflecting its 35% recovery rate. Similar results are shown for B and C. Observe that in each state for which A defaults (2, 5, 6, 8), the probability of A defaulting is shown as 5%. In all other states, the probability of A's payoff is 95%. The probabilities of B and C defaulting or paying off depend on what A does. For B, the probability of default is 7% if A does not default and 65% if A does default. Therefore, if A does not default, the probability of B not defaulting is 93%. If A defaults, the probability of B not defaulting is 35%. Similar logic is used for C. The joint probability in the last column is the product of the probabilities for A, B, and C. Although not shown, the sum of the probabilities in the last column is 100%.

These probabilities and the payoffs can then be used to determine what each bond is worth and what the overall CDO is worth. Multiplying the payoff of each bond in each outcome by the probability of that outcome for that bond gives us the expected payoff of the bond. Discounting these respective expected values gives the value of each bond. As

noted, we will use the 4% continuously compounded risk-free rate discount rate.<sup>9</sup> The values are

$$A : (\$1,000(0.8482) + \$350(0.0079) + \dots + \$1,000(0.0027) + \$350(0.0179))e^{-0.04} = \$968$$

$$B : (\$1,000(0.8482) + \$1,000(0.0079) + \dots + \$250(0.0027) + \$250(0.0179))e^{-0.04} = \$926$$

$$C : (\$1,000(0.8482) + \$1,000(0.0079) + \dots + \$1,400(0.0027) + \$150(0.0179))e^{-0.04} = \$944$$

The total of these three bond values is  $\$930 + \$889 + \$907 = \$2,726$ . This is the value of the CDO, as it reflects the value of the underlying bonds.

Now let us construct a table of the tranche payoffs. Consider the three tranches, S (senior), M (mezzanine), and E (equity). The sum of their payoffs is the total CDO payoff.

Outcome	Default Loss	Tranche Payoffs			
		S	M	E	CDO
1	\$0	\$800	\$1,000	\$1,200	\$3,000
2	\$650	\$800	\$1,000	\$550	\$2,350
3	\$750	\$800	\$1,000	\$450	\$2,250
4	\$850	\$800	\$1,000	\$350	\$2,150
5	\$1,400	\$800	\$800	\$0	\$1,600
6	\$1,500	\$800	\$700	\$0	\$1,500
7	\$1,600	\$800	\$600	\$0	\$1,400
8	\$2,250	\$750	\$0	\$0	\$750

First look at Outcome 8, in which all three bonds default for total losses of \$2,250. E is subject to a loss of \$800, M to a loss of \$1,000, and S to a loss of \$800. So E is allocated a loss of \$800, leaving a loss of \$1,450 to be allocated among M and S. Then M is then allocated a loss of \$1,000, leaving a loss of \$450 to be allocated to S. The total amount paid off (from the previous table) is \$750 so it goes entirely to S. In Outcomes 7, 6, and 5 the total losses exceed E's limit so E gets nothing, and M bears the remaining unallocated losses. In Outcome 5, for example, losses are \$1,400 of which \$1,200 is allocated to E, leaving \$200 to be absorbed by M, leaving M with a payoff of \$800. All

<sup>9</sup>We will not get into the issue of the appropriate discount rate. With the existence of credit derivatives, the credit risk can be hedged, thereby resulting in the ability to use the risk-free rate. The use of risk-free discount is permissible in markets in which risk can be hedged as discussed in other teaching notes and books.

of the default losses have then been absorbed so S gets its full \$800 payoff. This pattern can be followed through for each of the outcomes. Note that E loses the full amount of \$800 in Outcomes 5-8 and receives the full payoff of \$1,200, thus absorbing no losses, only in Outcome 1. In Outcomes 2, 3, and 4 E absorbs all of the losses but these losses are less than its limit. Thus, in Outcomes 2, 3, and 4, M absorbs no losses. M's only losses occur in Outcomes 5-8, in which E's limit cannot absorb all of the losses. S loses only in Outcome 8.

The tranche payoffs in each outcome are then multiplied by the probabilities of each outcome to get the expected payoffs

$$S : (\$800(0.8482) + \$800(0.0079) + \dots + \$800(0.0027) + \$750(0.0179))e^{-0.04} = \$768$$

$$M : (\$1,000(0.8482) + \$1,000(0.0079) + \dots + \$600(0.0027) + \$0(0.0179))e^{-0.04} = \$937$$

$$E : (\$1,200(0.8482) + \$550(0.0079) + \dots + \$0(0.0027) + \$0(0.0179))e^{-0.04} = \$1,022$$

The sum of these value is  $\$768 + \$937 + \$1,022 = \$2,726$ , the value we obtained as the value of the three bonds held in the CDO. The total value of the tranches must equal the value of the bonds. These tranche values are the amounts that must be invested to buy each of the respective tranches.

So we see that senior trancheholders invest \$768 and are subject to a loss of \$800, mezzanine trancheholders invest \$937 and are subject to a loss of \$1,000, and E trancheholders invest \$1,022 and are subject to a loss of \$1,200. Keep in mind, however, that these loss limits do not imply that the trancheholders can lose more than they invest. The loss limits merely reflect how much of the losses they absorb. Their minimum payoffs are zero, not a negative amount. Thus, for example, E invests \$1,022 but is subject to a loss limit of \$1,200. This means it must bear the first \$1,200 of credit losses. If credit losses on the portfolio are less than \$1,200, it bears the loss and is paid \$1,200 minus the loss it bears. If credit losses exceed \$1,200, the E trancheholders will assume \$1,200 in losses and will receive no payments. These points apply likewise to the mezzanine and senior trancheholders. Thus, their maximum payoffs are the loss limits.

The promised returns on each of these tranches can be determined by comparing the tranche loss limits to the amounts invested.

$$S : \frac{\$800}{\$768} - 1 = 0.0420$$

$$M : \frac{\$1,000}{\$937} - 1 = 0.0672$$

$$E : \frac{\$1,200}{\$1,022} - 1 = 0.1004$$

In other words, these are the returns that would be earned in the absence of any defaults. Notice that the promised returns are aligned according to the risk.

We can also obtain the probability that a given tranche will experience a default. For the senior tranche, we see that default occurs only in Outcome 8 so the probability that the senior tranche will experience a default is the probability of Outcome 8, which is 1.79%. For the mezzanine tranche, default is experienced in Outcomes 5-8. The sum of the probabilities of these tranches is 1.46% + 0.96% + 0.27% + 1.79% = 4.48%. For the equity tranche, default occurs in every outcome except the first. Thus, the probability that the equity tranche suffers a default loss is one minus the probability of the first outcome, or 1 – 84.82% = 15.18%.

The correlation of defaults has a major effect on the pricing of the tranches. If the underlying bonds are more highly correlated in the sense that if one defaults, the other is more likely to default, then the risk to the mezzanine and senior tranches is higher. In other words, if it is more likely that if one bond defaults, others will default, then it becomes more likely that the equity tranche will be unable to bear all of the default losses. This increases the risk for the mezzanine tranches and possibly for the senior tranches.

### **Variations**

Of course, most CDOs have many more tranches than we assumed here and as noted, many tranches of different priorities can exist inside a given class, such as the mezzanine class.<sup>10</sup> In addition, the allocation of losses can follow a far more complicated schedule than what we assumed in this example. For example, losses can be allocated to the equity, mezzanine, and senior tranches without allocating all of the losses. The remaining losses then go back to the equity, mezzanine, and senior tranches in that order. In other words, the mezzanine and senior trancheholders might bear losses before the

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<sup>10</sup>There is typically only one equity tranche.

equity trancheholder hits its limit. Clearly in such a case, the equity tranche is less risky and the mezzanine and senior tranches are riskier than in the more standard case. Of course, these features are priced into the tranches and into the overall CDO. In this example, the tranches were close in size but that need not be the case. Also, in practice most of the underlying bonds or loans make interim interest and sometimes principal payments, but these payments are allocated in the same manner as shown above.<sup>11</sup> Some CDOs are actively managed in the sense that the component bonds can be traded and other CDOs are static, meaning that the instruments in the CDO are held for the life of the CDO. Some CDOs hold other CDOs and are appropriately called CDO<sup>2</sup> (CDO Squared). It is even possible to not have an equity tranche.<sup>12</sup> Another variation of the CDO is the synthetic CDO in which the underlying bonds are generally default-free, such as U. S. treasury bonds. Credit is then added by selling credit default swaps that cover other bonds that are default-free.

### **Appendix: Determining the Probabilities of Default of Bonds B and C**

Recall that the information we have about the probabilities of default is as follows:

<b>Event</b>	<b>Probability</b>
A defaults	5%
B defaults, given that A defaults	65%
B defaults, given that A does not default	7%
C defaults given that A defaults	55%
C defaults given that A does not default	4%

We are given that the probability that A default is 5%. We might be interested in the probability that B defaults and the probability that C defaults. That information is not in the above table but can be determined. For B, we have

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<sup>11</sup>When the underlying securities make coupon payments, the CDO tranches are promised a certain rate of coupon payments, though of course the actual payments can be less due to defaults.

<sup>12</sup>In that case, there is considerably greater risk to the mezzanine and senior tranches. The equity tranche exists if the CDO is said to be *overcollateralized*, meaning that the value of the assets in the portfolio exceeds the value of the non-equity tranches issued. Thus, the equity serves as excess collateral and clearly helps the mezzanine and senior trancheholders.

Probability that B defaults given that A defaults = 65%

Probability that B defaults given that A does not default = 7%

By definition,

Probability that B defaults =

Probability that B defaults given that A defaults

\* Probability that A defaults

Plus

Probability that B defaults given that A does not default

\* Probability that A does not default

This is

$$0.65 * 0.05 + 0.07 * 0.95 = 0.099,$$

That is, 9.9%. For C,

$$0.55 * 0.05 + 0.04 * 0.95 = 0.0655,$$

That is 6.55%. Thus, the probability that A will default is 5%, the probability that B will default is 9.9%, and the probability that C will default is 6.55%.